Pre-Algebra

Know basic fractions and conversions to decimals and percents (and vice versa) ½ = 0.5 = 50% 1/3 = .33 = 33 1/3 % ¼ = .25 = 25% 1/8 = .125 = 12.5%
Know the perfect squares (from 0 to 625) **Know the perfect square roots** (from 0 to 25)
Ordering Numbers Least to Greatest or Greatest to Least **Convert** fractions to decimals, if that is easier for you. **Watch the signs**.
Absolute Value The distance a number is from 0 on the number line. Since distance is always positive, a pure absolute value has a positive result.
Real Numbers and Its Subsets Natural Numbers Whole Numbers Integers Rational Numbers Irrational Numbers
Solving One and Two Step Equations Remember to use inverse operations carefully.
Probability The ratio of "successful" to "total" **Odds** The ratio of "successful" to "unsuccessful" (Successful + Unsuccessful = Total)
The Fundamental Counting Principle Two independent events, called *a* and *b*, can occur $a \cdot b$ ways. Sometimes a **tree diagram** can help you to see the total number of possible events.

Charts, Tables, Graphs

Read the title and the labels on each axis carefully.

Sometimes the questions are asking for a total or an average in a particular part of the table or chart.

Be able to distinguish between a relation and a function by looking at a graph.

Vertical Line Test is a visual way to identify a function vs a relation.

Mean, Median, Mode

All of these are "central measures of tendency." Each has its particular use.

Mean is the average: Add up the numbers and divide by the number of numbers.

Median is the middle number of a sorted set of numbers. (least to greatest or greatest to least – doesn't matter.)

Mode is the most frequently occurring number in a set of numbers.

Geometry

Geometry is the study of measurement, congruence, and similarity. For the study of measurement, we need to have a toolbox of formulas, including those for length, perimeter, circumference, area, and volume. A handout is available with lots of formulas.

For congruence of two triangles, we use postulates like SSS, SAS, ASA, and AAS. (There is not a congruence postulate for SSA.)

Remember that corresponding parts of congruent objects are congruent. If a congruence statement is given, match the corresponding parts to find missing measures.

For similarity of objects, remember that corresponding angles are congruent and corresponding sides are proportional. If a similarity statement is given, match the corresponding parts to set up an equation (for angles) and a proportion for corresponding sides.

Triangles

- The sum of the measures of any two sides of a triangle must be greater than the measure of the third side.
- Opposite the smallest angle is the shortest side.
- Opposite the largest angle is the longest side.
- Opposite the shortest side is the smallest angle.
- Opposite the longest side is the largest angle.

Consider two triangles with a pair of sides in one triangle congruent to the corresponding sides in the other triangle

- The measure of the angle between each pair determines the length of the third side

The Pythagorean Theorem

In a right triangle, the sum of the squares of the legs, a and b, equals the square of the hypotenuse, c.

$$a^2 + b^2 = c^2$$

It's a good idea to have the other forms of the Pythagorean Theorem in your list of memorized formulas.

$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$

The Pythagorean Theorem converts to circular functions: $x^2 + y^2 = r^2$

$$r = \sqrt{x^2 + y^2}$$
$$x = \sqrt{r^2 - y^2}$$
$$y = \sqrt{r^2 - x^2}$$

Special Right Triangles

Special Ratios of the Sides of the Triangle

30° -- 60 ° -- 90°

$$1 : \sqrt{3} : 2$$

45° -- 45° -- 90°

$$1 : 1 : \sqrt{2}$$

Pythagorean Triples

Pythagorean triples are measures of the sides of a triangle that guarantee a right triangle. Here's a list of commonly used triples. Knowing these can save time when you have a missing measure. Remember that the largest number represents the length of the hypotenuse.

$$3, 4, 5$$
$$5, 12, 13$$
$$7, 24, 25$$
$$8, 15, 17$$
$$9, 40, 41$$

Polygons

Sum of the Measures of the Interior Angles of a Polygon (in degrees)

Let n = the number of sides

$(n - 2)180°$

Sum of the Measures of one set of Exterior Angles of a Polygon (in degrees)

360°

Regular Polygon

All angles are congruent.

All sides are congruent.

Measure of <u>one</u> interior angle of a <u>regular</u> polygon

$$\frac{(n-2)180°}{n}$$

Measure of <u>one</u> exterior angle of a <u>regular</u> polygon

$$\frac{360°}{n}$$

Number of diagonals in a polygon

$$\frac{(n-3)n}{2}$$

Area of a regular polygon

$$A = \frac{1}{2}Pa$$

P = the **perimeter** of the polygon

a = the length of the **apothem**

A **regular** polygon has a **center**.

The **apothem** is the segment from the center to the midpoint of a side.

The **radius** is the segment that **bisects** a vertex.

Circles

A circle is the set of all points equidistant from one point, called the center. A segment from the center of the circle to any point on the circle is called the radius, r. The longest **chord** in a circle is the **diameter**, d, the chord through the center of the circle. Knowing the formulas for circumference and area are very important.

Equation of a Circle (in a coordinate plane, with center (h,k) and radius r.) $(x-h)^2+(y-k)^2=r^2$
There are 360° (or 2π **radians**) in one revolution around a circle.
Circumference $C=2\pi r$ or $C=\pi d$ (since $d=2r$) **Area** $A=\pi r^2$
Arc Length The measure of the length around the complete circle is called the circumference. The measure of part of the circumference is called *arc length*, s. $s=\frac{n}{360}\cdot C$ $s=\frac{n}{360}\cdot 2\pi r$ $s=\frac{n}{360}\cdot \pi d$ $n=$ the degrees of the intercepted arc
Area of a Sector A **sector** is part of the area of a circle. We use K to denote **area of a sector.** $K=\frac{n}{360}\cdot A$ $K=\frac{n}{360}\cdot \pi r^2$ $n=$ the degrees of the intercepted arc

Trigonometry

Most trigonometry courses begin with "right triangle trigonometry." The trigonometry terms called the sine, cosine, or tangent of an acute angle in a right triangle are the ratios of two sides of a right triangle.
Consider a right triangle, with right angle C.
Angles A and B are the acute angles.

- Label the hypotenuse first. (It's the longest side – and has to be, because it's the side across from the right angle, the largest angle in the right triangle.
- Determine and label which side is opposite the acute angle that you are considering. (It's the side that does not form the angle.)
- The adjacent side is the second side that forms the angle. (The hypotenuse is the other side that forms the acute angle.

$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp} \qquad\qquad \sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj} \rightarrow \text{converts to circular functions} \rightarrow \cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp} \qquad\qquad \tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The **Law of Sines** is a proportion that we use to solve oblique triangles. (Oblique triangles are triangles that are not right triangles.} So, if the given information is three of the four parts of a proportion, use the Law of Sines to find the missing information. For multiple choice questions, working from the one full ratio backwards is a smart technique.

Law of Sines, Ambiguous Case (SSA)

If the given information is two consecutive sides and the non-included angle, we must check for one of three cases. Either there is no triangle, one triangle or two triangles. Handout with a complete explanation is available. Again, for multiple choice questions, working from the one full ratio backwards is a smart technique.

Law of Cosines

We use the Law of Cosines to solve an oblique triangle when we cannot use the Law of Sines. This means that the given information does not give us three of the 4 parts of a proportion. In other words, use the Law of Cosines if the given information is SSS or SAS. It is acceptable to use the Law of Cosines for the first step of the problem, then change to the Law of Sines for the rest of the problem. As before, for multiple choice questions, **you can use one ratio from the Law of Sines to work backwards.**

Law of Cosines

To find the measure of a **side**... $a = \sqrt{b^2 + c^2 - 2bc\cos A}$

To find the measure of an **angle**... $A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right)$

Algebra Concepts

Functions

A function is a relation in which each element in the domain is paired with exactly one element in the range. Graphically, this means that any vertical line will intersect the graph at exactly one point.

Linear Functions

$y = x$

Domain: (all real numbers)

Set notation: $\{x \mid x \in \Re\}$

Interval notation: $(-\infty, \infty)$

Range: (all real numbers)

Set notation: $\{y \mid y \in \Re\}$

Interval notation: $(-\infty, \infty)$

Absolute Value Functions

The absolute value function relates to "finding the absolute value of a number," which is distance from zero on the number line.

$y = |x|$

Domain: (all real numbers)

Set notation: $\{x \mid x \in \Re\}$

Interval notation: $(-\infty, \infty)$

Range: (zero, and all positive numbers)

Set notation: $\{y \mid y \geq 0\}$

Interval notation: $[0, \infty)$

Square Root Functions

In the real number system, the smallest number that we can "take the square root of" is the number 0. This defines the domain. Then, because we are taking the square root of 0 and positive numbers, the output will only be 0 and positive numbers. This defines the range. $y=\sqrt{x}$

Domain: (zero, and all positive numbers)

Set notation: $\{x \mid x \geq 0\}$

Interval notation: $[0, \infty)$

Range: (zero, and all positive numbers)

Set notation: $\{y \mid y \geq 0\}$

Interval notation: $[0, \infty)$

Rational Functions

A rational function contains a variable in the denominator. We cannot divide by zero, so we must exclude all values that make the denominator equal to zero. This defines the domain. Because a rational function does not have a numerator of zero – we wouldn't have a rational function, were that the case – zero is excluded from being a possible output. This defines the range.

$$y=\frac{1}{x}$$

Domain: (all real numbers except zero)

Set notation: $\{x \mid x \neq 0\}$

Interval notation: $(-\infty, 0) \cup (0, \infty)$

Range: (all real numbers except zero)

Set notation: $\{y \mid y \neq 0\}$

Interval notation: $(-\infty, 0) \cup (0, \infty)$

Finding the Equations of Asymptotes

(or the coordinates of holes)

In Rational Functions

Vertical Asymptote	**Horizontal Asymptote**	**Slant Asymptote**	**Hole**
Hint: The denominator cannot equal zero.	Hint: Compare the degree of the numerator with the degree of the denominator.	Hint: Use long division to find the equation of a line. $y=mx+b$	Hint: If the denominator is a factor of the numerator, there is a hole at that x-value.
Suggestion: Check for common factors before assuming there is a V.A.	Suggestion: Do "<, =, >" in that order because it will help you if there is a S.A.	Suggestion: Check for common factors and if the degree of the numerator is 1 more than the denominator.	Suggestion: Find the y-value of the ordered pair of the hole after simplifying the function.
Equation is of the form x = ____	Equation is of the form y = ____	Equation is of the form y = mx + b – so only complete 2 steps in long division and ignore the remainder.	Coordinates of the form (x, y)

Long Division/Synthetic Division of Polynomials

The most important concept to remember is that all powers of x must be represented in the dividend (the expression in the numerator) in both long division and in synthetic division.

Another concept to remember is that a remainder of 0 indicates that the divisor is a factor. This helps us to determine zeroes in the graph.

Inequalities

Things to remember:
1. Solve for y.
2. Dashed line when there is NOT an equal sign in the inequality symbol
3. When multiplying or dividing by a negative number, change the direction of the inequality sign.
4. When determining how to shade a graph of an inequality, look at the y-intercept. This will help you to see that "greater than" is above that point and "less than" is below that point.

Matrices

$m \times n$

(rows x columns)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\text{determinant } A = \det A = ad - bc$$

Scalar multiplication multiplies each term by the same number.

Exponent Rules

1. $a^m \cdot a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $a^0 = 1$
5. $a^m = a^n \Leftrightarrow m = n$

Logarithms—Common Logarithms/Natural Logarithms

To understand logarithms, remember these two things: (1) a logarithm is an exponent and (2) exponential functions and logarithm functions are inverse functions.
Refer to this simple example whenever you have a block in your understanding.

$$y = a^x \rightarrow \log_a y = x$$

So, when you see something like $2^3 = 8 \rightarrow \log_2 8 = 3$. Think of other examples to "make this stick." When you see a problem that looks like one of these, it's easy to convert to find a missing value.

$\log_x 64 = 3$ converts to $x^3 = 64$ and then we solve for x. $(x = 4)$

$\log_6 216 = x$ converts to $6^x = 216$ and then we solve for x. $(x = 3)$

$\log_5 x = 3$ converts to $5^3 = x$ and then we solve for x. $(x = 125)$

Then, the next concepts to understand are the properties of logarithms. Remember that the properties for general logarithms (any base), for common logarithms (base 10) and for natural logarithms (base e) are identical. Remember that the base must be a positive number, not equal to 1.

Rules of Logarithms

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^y = y \log_a x$
4. $\log_a 1 = 0$
5. If $\log_a m = \log_a n$, then $m = n$.

Sequences and Series

Arithmetic Sequences

$$t_n = a + (n-1)d$$
$d =$ common difference
$n =$ the n^{th} term
$a =$ the first term

Geometric Sequences

$$t_n = ar^{n-1}$$
$a =$ the first term
$r =$ common ratio
$n =$ the n^{th} term

Arithmetic Series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Geometric Sequences

$$S_n = \frac{a - ar^n}{1 - r}, r \neq 1$$